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OPTIMUM DESIGN OF STIFFENED SHEAR WEBS WITH
SUPPLEMENTARY SKIN STABILIZATION

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20. Abstract

This report documents the result of an investigation of the optimum design of shear web structures whose function is to resist buckling. The structure considered was comprised of a simply supported thin metal web of constant depth, but infinite length, stabilized at uniform intervals by vertical stiffeners. The analysis was based on that of Stein and Fralich, and Cook and Rockey, and takes proper account of individual stiffener action in the buckling process and utilizes a realistic buckling mode between stiffeners.

Results indicated that structural weight decreases continuously as stiffener spacing was reduced until an absolute minimum was achieved at a spacing well below practical limits. Computer routines based on the analysis have been written.

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SUMMARY

The optimum design of stiffened shear webs is considered.

A design procedure is developed, based on the analysis of Stein and Fralich[8], supplemented by an orthotropic plate solution due to Timoshenko[11].

The effect of adding a layer of low density material to the skin between stiffeners is included.

The results show that the lightest designs require very small stiffener spacing.

Substantial improvements in efficiency are indicated when supplementary skin stabilisation is utilised.

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1.   Introduction

The problem of designing shear webs has been studied by many authors including notably Kuhn[1,2] Symonds[3] Shanley[4] and Rothwell[5] and more recently, Parsons and Beard[6], and Laakso and Strayer[7]. However it has rarely been possible to generalise the results obtained sufficiently to allow direct application by the structural designer.

This report is concerned with the optimum design of shear webs of the type shown in figure 1.

The structure, which is required to resist buckling under the design loading, comprises a simply supported thin metal web of constant depth but infinite length, stabilised at uniform intervals by vertical stiffeners. The web between the stiffeners may be further stabilised by the addition of a layer of rigid organic foam. The structure is assumed to be linearly elastic up to the design load.

The analysis of buckling is based on that of Stein and Fralich[8], and Cook and Rockey[9] which is incorporated in the Engineering Sciences Data Sheets[10] widely used by structural analysts and designers. This analysis takes proper account of individual stiffener action in the buckling process, and utilises a realistic buckling mode between stiffeners.

Comparison has been made with designs based on Timoshenko's[11] orthotropic plate buckling theory which assumes the effect of stiffener stiffness to be smeared uniformly over the web. This method is found to be most appropriate as stiffener spacing tends to zero, whereas buckling strength will be over estimated progressively as stiffener spacing increases.

2.   Analysis

2.1   Buckling analysis with finite stiffener spacing

An infinitely long shear web of depth a^*, thickness t^* is to be designed to resist an applied shear flow q^* without buckling.

The web is subdivided into panels by a series of vertical stiffeners spaced a distance c^* apart along the web.

The buckling of the web may be characterised in terms of individual panel dimensions by the following relationship

$$q^*_{crit} = KE^* \frac{t^{*3}}{c^{*2}} \qquad \ldots (1)$$

where E^* is the web material elastic modulus.

The buckling coefficient K depends on the panel boundary conditions and the panel proportions, given by

$$c = \frac{c^*}{a^*} \qquad \ldots (2)$$

The panels will be assumed simply supported at the long edges of the web.

The remaining boundaries are provided by the stiffeners, whose influence depends upon the stiffener bending stiffness parameter

$$\mu = \left[\frac{E^*_s I^*_s c^*}{E^* a^{*2} t^{*3}} \right]^{\frac{1}{4}} \qquad \ldots (3)$$

where I_s = stiffener effective second moment of area.

E_s = stiffener material elastic modulus.

Reference 9 gives the buckling coefficient K as a function of c and μ in graphical form.

From equation (1), the skin thickness required to prevent buckling due to the shear flow q^* is

$$t^* = \left[\frac{c^* q^*}{KE^*} \right]^{\frac{1}{3}} \qquad \ldots (4)$$

From equation (3), stiffener bending stiffness required is

$$E_s^* \, I_s^* \;=\; \mu^2 \frac{E^* a^{*2} t^{*3}}{c^*}$$

or, if stiffener and web are made from the same material

$$I_s^* \;=\; \frac{\mu^2 a^{*2} t^{*3}}{c^*} \qquad \ldots (5)$$

By assuming a linear relationship between stiffener bending stiffness and stiffener area, a dimensionless stiffener shape factor ϕ_S may be determined for any given family of stiffener designs, so that

$$I_s^* \;=\; \phi_S a^{*2} A_s^* \qquad \ldots (6)$$

Equation (6) would apply, for instance, to a range of thin-walled stiffeners of constant size, but with varying wall thickness.

The equivalent web thickness attributable to the stiffeners may now be found from (5) and (6) which give

$$t_s^* \;=\; \frac{A_s^*}{c^*} \;=\; \frac{\mu^2 t^{*3}}{\phi_S c^{*2}}$$

so that from (4)

$$t_s^* \;=\; \frac{\mu^2 q^*}{K \phi_S E^*} \qquad \ldots (7)$$

An expression for total equivalent thickness may now be written in the following form

$$T_e \;=\; \frac{t_e}{Q^{\frac{1}{3}}} \;=\; \frac{1}{\eta}\left[\frac{c^2}{K}\right]^{\frac{1}{3}} + R.\frac{\mu^2}{K} \qquad \ldots (8)$$

where $\;\; t_e \;=\; \dfrac{t_e^*}{a^*} \;=\; \dfrac{t^*}{a^*} + \dfrac{t_s^*}{a^*}$

$$Q = \frac{q*}{a*E*} \qquad \ldots (9)$$

$$\text{and} \quad R = \frac{Q^{\frac{2}{3}}}{\phi_s} \qquad \ldots (10)$$

The quantity η is the skin efficiency factor which will be defined in appendix A for the foam stabilised web. If the panels between stiffeners are unstabilised, i.e. without additional foam, $\eta = 1$.

Using equation (8), for any given value of the parameter R the quantities c and μ may be varied to determine their optimum values, so that the total equivalent thickness is minimised.

Since the buckling coefficient K is available as a function of c and μ only in numerical form, the optimisation process will inevitably involve numerical interpolation. However, experience shows equation (8) to be well-behaved so that the required results may be obtained numerically with excellent consistency and accuracy.

A Fortran routine has been developed which utilises the available numerical data to evaluate equation (8).

For given values of load intensity R and web efficiency η the programme determines values of the equivalent thickness parameter T_e as a function of c and μ, and then proceeds to identify the locus of minimum T_e values.

Data generated by this programme is shown plotted in figures 2 to 4. A second version of the programme is available which operates entirely with dimensional quantities. Both programmes are described in Appendix B.

2.2 Orthotropic plate analysis

The design procedure described below is based on the analysis of Timoshenko and Gere[11], page 407, which assumes web bending stiffness to be uniform at all points of the plate, although having different values in direction parallel and perpendicular to the plate edges.

Reference 11 gives for the plate critical shear flow, ...

$$q^*_{crit} = \frac{4k}{a^{*2}}(D_1 D_2^3)^{\frac{1}{4}} \qquad \ldots (11)$$

where D_1 = plate bending stiffness perpendicular to stiffeners

$$= \frac{E^* t^{*3}}{12} \qquad \ldots (12)$$

D_2 = plate bending stiffness parallel to stiffeners

$$= \frac{E^*_s I^*_s}{c^*} \qquad \ldots (13)$$

k = buckling coefficient depending on parameters β, θ.

$$\beta = \frac{a^*}{b^*}\left[\frac{D_1}{D_2}\right]^{\frac{1}{4}}$$

$$\theta = \frac{(D_1 D_2)^{\frac{1}{2}}}{D_3} \qquad \ldots (14)$$

b^* = overall web dimension perpendicular to stiffeners ($= \infty$ in this case)

D_3 = plate twisting stiffness

$$= \frac{G^* t^{*3}}{6} \qquad \ldots (15)$$

When $\beta = 0$, the following equation fits the data given by Timoshenko in figure 9-42 with good accuracy

$$k = 8 + \frac{5}{\theta} \qquad \ldots (16)$$

Equations (12) to (16) may now be substituted into (11) so that, after some reorganisation, the following cubic equation is found

$$(TI_s^{\frac{1}{4}})^3 + \frac{5\Gamma^2}{16\sqrt{3}(1+\nu)}(TI_s^{\frac{1}{4}}) - \frac{12^{\frac{1}{4}}}{32} = 0 \qquad \ldots (17)$$

where $I_s = \frac{I^*_s}{c^* t^{*3}}$; $T = \frac{t}{Q}\frac{1}{3}$ $\qquad \ldots (18)$

Equation (17) is in Cardan's form, $x^3 + rx + s = 0$, for which the following root exists

$$x = \left[-\frac{s}{2} + \left(\frac{s^2}{4} + \frac{r^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[-\frac{s}{2} - \left(\frac{s^2}{4} + \frac{r^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad \ldots (19)$$

where

$$x = TI_s^{\frac{1}{3}}$$

$$r = \frac{5T^2}{16\sqrt{3}(1+\nu)}$$

$$s = -\frac{12^{\frac{1}{3}}}{32}$$

It may be demonstrated that for the range of values involved in the current problem

$$\frac{r^3}{27} << \frac{s^2}{4}$$

so that (19) simplifies to give

$$I_s^{\frac{1}{3}} T = \left(\frac{12^{\frac{1}{3}}}{32}\right)^{\frac{1}{3}} = p \quad \ldots (20)$$

The total equivalent plate thickness may be formed by using equations (6), (9), (10) and (18) to give

$$T_e = T + \frac{R}{T}(I_s^{\frac{1}{3}} T)^4$$

which, using (20) gives

$$T_e = T + \frac{p^4 R}{T} \quad \ldots (21)$$

The value of T to give minimum T_e may be found from (21) by differentiation, which process produces the following results.

$$T_o = T_{so} = p^2\sqrt{R} \quad \ldots (22)$$

$$T_{eo} = 2p^2\sqrt{R} \quad \ldots (23)$$

$$I_{so} = \frac{1}{p^2 R} \quad \ldots (24)$$

A range of optimum designs of equal merit lie on a ray in $\mu-c$ space given by

$$c = p^2 R\mu \quad \ldots (25)$$

Expressed in dimensional quantities, the equivalent shear stress for optimum designs is given by

$$\tau_{eo}^* = \frac{\phi_s^{\frac{1}{2}}}{2p^2} \left[\frac{q^* E^*}{a^*}\right]^{1/3} \qquad \ldots (26)$$

and since $T_{so} = T_o$, the true stress in the web is

$$\tau_o^* = 2\tau_{eo}^* \qquad \ldots (27)$$

The remainder of the μ-c space may be mapped by noting from (3) and (18) that

$$I_s = (\frac{\mu}{c})^2$$

so that from (20)

$$T = p(\frac{c}{\mu})^{\frac{1}{2}} \qquad \ldots (28)$$

This may be substituted into (21) which may then be solved to give an equation for the equivalent thickness contours in the following form.

$$c = \left[\left(\frac{T_e}{2p}\right) \pm \left\{\left(\frac{T_e}{2p}\right)^2 - p^2 R\right\}^{\frac{1}{2}}\right]^2 \cdot \mu \qquad \ldots (29)$$

The lines are rays from the origin, the upper and lower roots giving contours on either side of the optimum given by equation (25). A typical solution is illustrated in figure 5. Care must be exercised in applying equation (29) to the design process. Orthotropic plate theory has been shown to apply reliably only when stiffeners are very closely spaced (see reference 8). Thus equation (29) should be regarded as simply providing a guide to behaviour in the extreme regions of the design charts, as shown in figures 2 to 4.

3. Discussion

Certain consistent features may be observed from the results based on equation (8), which are illustrated in figures 2,3,4 and 9 It would appear that for most practical purposes, weight reduced continuously as stiffener spacing is reduced.

This process reaches its limit at very small stiffener spacings indicated by the orthotropic plate solution in equations (22) to (25). This solution appears to be consistent with that based on the more accurate analysis which takes account of finite stiffener spacing in the buckling mode. Numerical results based on the more sophisticated analysis are available only for values of c greater than 0.2, and therefore this has been chosen as a point of demarcation between the solutions.

The addition of low density material to plate elements offers a substantial inprovement in efficiency, as indicated in figure 8. Figure 10 indicates that these improvements may be successfully incorporated in shear webs of the type under consideration.

One way in which these benefits may be realised is to utlise a higher web efficiency to allow larger stiffener spacing without weight penalty, thus producing a more practical design.

4. Conclusions

An analysis has been developed which leads to optimum designs for long shear webs with simply supported edges stabilised ⌐ a regular array of vertical stiffners. Stress levels are ssumed to be below the elastic limit, and a linear relationship between stiffener area and second moment of area is implied.

The effect of additional low density material applied to the skins between stiffners has also been studied.

The results indicate that structure weight decreases continuously as stiffener spacing is reduced until an absolute minimum is achieved at a spacing well below practical limits.

The benefits to be obtained from the addition of low density elements are found to be substantial.

Computer routines based on the analysis have been written. These programmes may be used as a basis for further parametric studies (SHEA) and for practical design purposes (SWEB).

5. Principal notation

a^*	Web depth
c^*	Stiffener spacing
t^*	skin thickness
t_s^*	stiffener equivalent skin thickness
t_e^*	total equivalent skin thickness
A_s^*	stiffener area
I_s^*	stiffener second moment of area
q^*	shear flow
E^*	Young's modulus
G^*	shear modulus
τ^*	shear stress
ρ^*	density

$$I_s = \frac{I_s^*}{c^* t^{*3}}$$

$$c = c^*/a^*$$

$$t = t^*/a^*$$

$$\mu = c\, I_s^{\frac{1}{3}}$$

$$Q = \frac{q^*}{a^* E^*}$$

$$\phi_s = \frac{I_s^*}{A_s^* a^{*2}}$$

$$R = Q^{\frac{2}{3}}/\phi_s$$

$$T = t/Q^{\frac{1}{3}}$$

Z = plate bending stiffness parameter (equation A3)

η = skin efficiency factor

$D_{1,2,3}$ = orthotropic plate bending stiffnesses

β, θ = orthotropic plate parameters

k = orthotropic plate buckling coefficient

K = skin buckling coefficient

$$p = \left(\frac{12^{\frac{1}{3}}}{32}\right)^{\frac{1}{3}}$$

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Appendix A

The Effect of a Secondary Stiffening Layer on Plate Buckling

Consider a thin flat plate of uniform thickness with given boundary conditions.

The edge loading at which such a plate will buckle is given by equation (1) which may be re-written

$$q^*_{crit} = \frac{12K}{c^{*2}} \cdot (E^*I^*) \qquad \ldots (A1)$$

where $I^* =$ plate second moment of area per unit width

$$= \frac{t^{*3}}{12}$$

We may now consider a similar plate composed of two layers of isotropic material characterised by suffices 1 and 2, the two materials having a common value of Poisson's ratio but being otherwise different. By assuming a linear distribution of bending strain through the thickness of the combined plate the following expression may be derived

$$(E^*I^*) = \frac{ZE_1 t_1^3}{12} \qquad \ldots (A2)$$

where

$$Z = 1 + \frac{E_2}{E_1}\left(\frac{t_2}{t_2}\right)^3 + \frac{3\frac{E_2}{E_1} \cdot \frac{t_2}{t_1}\left(1 + \frac{t_2}{t_1}\right)^2}{\left(1 + \frac{E_2}{E_1} \cdot \frac{t_2}{t_1}\right)} \qquad \ldots (A3)$$

If equations A2 and A3 are substituted into A1, the thickness t_1^* of material 1 may ve found which is required to prevent buckling under a given loading q^* in the following form

$$t_1^* = \left[\frac{q^* c^{*2}}{KZE_1}\right]^{1/3} \qquad \ldots (A4)$$

The weight of the plate per unit area is given by

$$W^* = \frac{\rho_1}{\eta} \left[\frac{q^* c^{*2}}{K E_1} \right]^{1/3} \qquad \ldots (A5)$$

where

$$\eta = \frac{z^{1/3}}{\left[1 + \frac{\rho_2^*}{\rho_1^*} \cdot \frac{t_2^*}{t_1^*} \right]} \qquad \ldots (A6)$$

$$\rho_{1,2}^* = \text{respective material densities}$$

When plate 2 is absent, $t^* = 0$ and $\eta = 1$, and so η, which is a function of

$$\frac{\rho_2^*}{\rho_1^*} \; ; \; \frac{E_2^*}{E_1^*} \quad \text{and} \quad \frac{t_2^*}{t_1^*} \quad \text{represents the efficiency of}$$

the combination, relative to the single layer plate of material 1.

$$\text{As } \frac{t_2^*}{t_1^*} \to \infty, \quad \eta \to \frac{\rho_1^*}{\rho_2^*} \cdot \left(\frac{E_2^*}{E_1^*} \right)^{1/3} = \eta_2 \qquad \ldots (A7)$$

η_2 is the relative efficiency of a plate composed entirely of material 2. Equation (A7) indicates why low density materials are superior for stability-critical structures, since for most metals

$$E^* \, \alpha \, \rho^*$$

$\therefore$ from (A7)

$$\eta_2 = \left[\frac{\rho_1^*}{\rho_2^*} \right]^{2/3} \qquad \ldots (A8)$$

If material 2 is a low density organic foam and material 1 is a metal, ρ_2^* will be an order of magnitude less than ρ_1^* and although usually for this combination of material

$$\frac{E_2^*}{E_1^*} < \frac{\rho_2^*}{\rho_1^*}, \text{ potential efficiency is high.}$$

Much of this gain in efficiency may be achieved without an excessive thickness of foam, as is shown in figure 7.

The resulting structure is simple to manufacture, with few detail design problems.

The above analysis applies to the design of any thin plate element which is stability critical under any given combination of edge loading.

Thus the results may be applied to the plate elements between the stiffeners of the shear webs which are the subject of this report.

The substantial benefit to be obtained from secondary stiffening in shear webs of this type is illustrated in figures 9 and 10.

Appendix B

Computer Programmes for Shear Web Design

Two Fortan programmes are available SHEA and SWEB, which are based on equation (8).

SHEA

This programme works entirely in terms of non-dimensional quantities and is suitable either for terminal operation or batch processing. Input requires two numbers only, the load intensity parameter R and skin efficiency factor, η.

Optimum solutions are given for a range of stiffener spacings. Since design continuously improves as stiffener spacing is reduced, the designer must choose from this list the best design available, which will be limited either by the smallest acceptable stiffener spacing, or the smallest acceptable stiffener size.

If required, a complete carpet of designs may be output as an aid to parametric studies.

Run times depend on installation, but are brief.

SWEB

This programme is similar to SHEA, but uses dimensional data for more direct design applications.

Input includes shear force, web depth, Young's Modulus, a sample stiffener area and corresponding second moment of area, and skin efficiency factor.

Output which is dimensional, is restricted to optimum designs, but includes information on web stresses. Consistent units are required throughout.

Listings and/or card decks are available from the author.
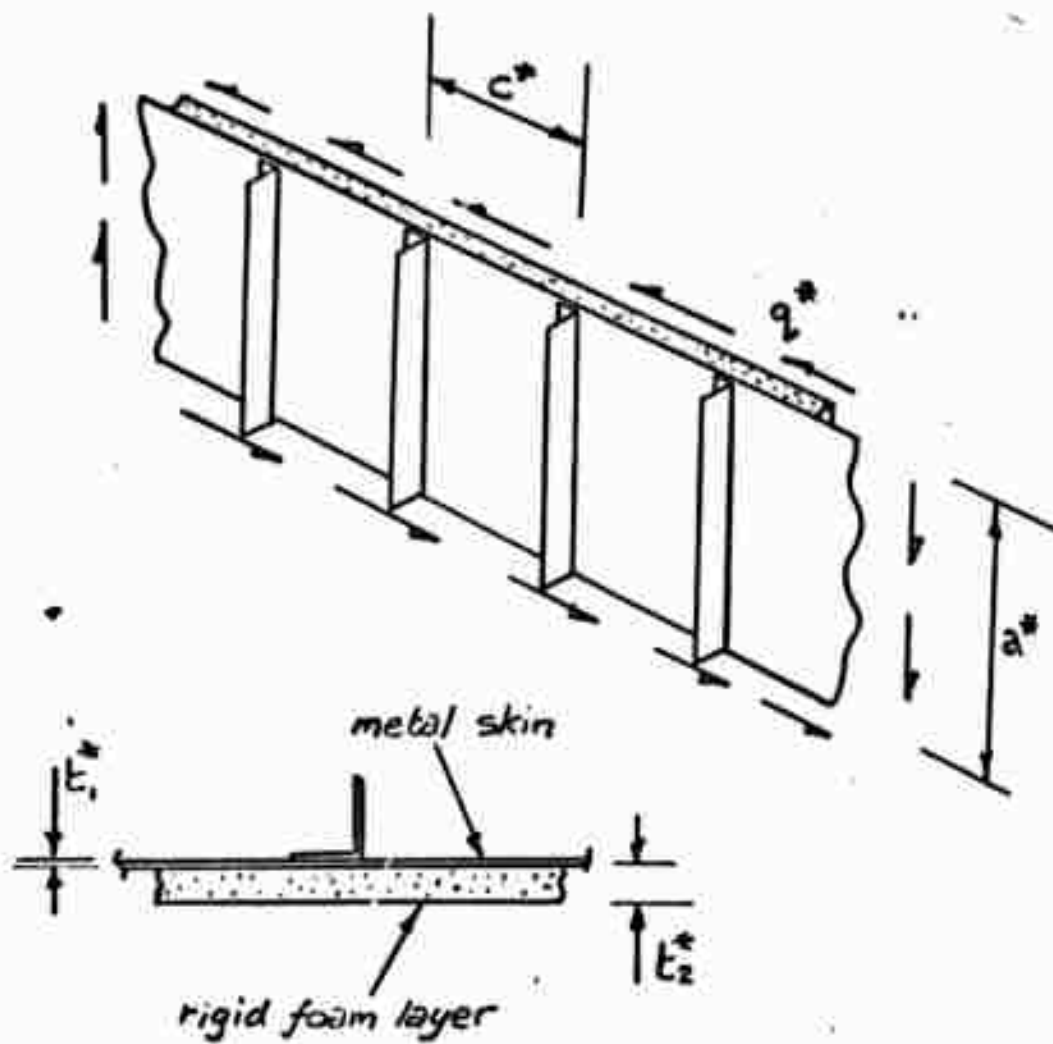
FIGURE 1.   Stiffened shear web with organic foam skin stabilisation.
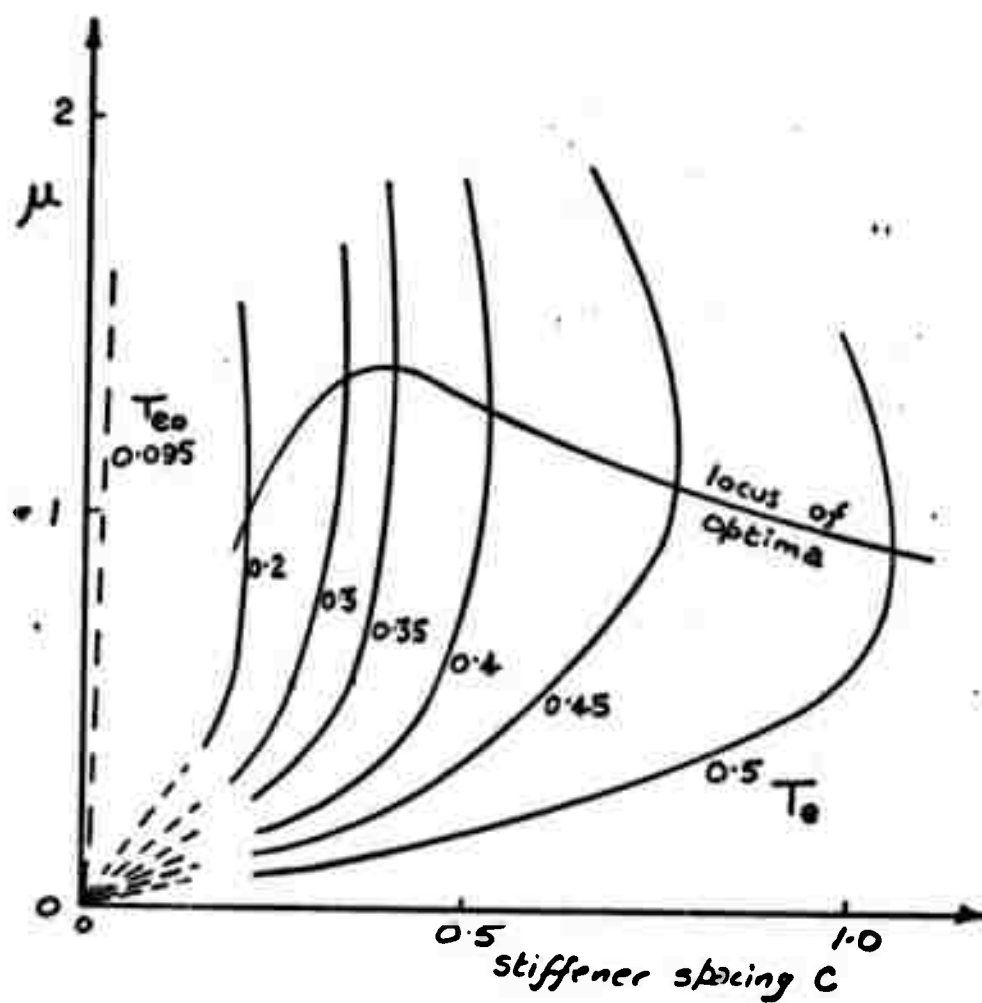
FIGURE 2. Shear web equivalent thickness related to stiffener
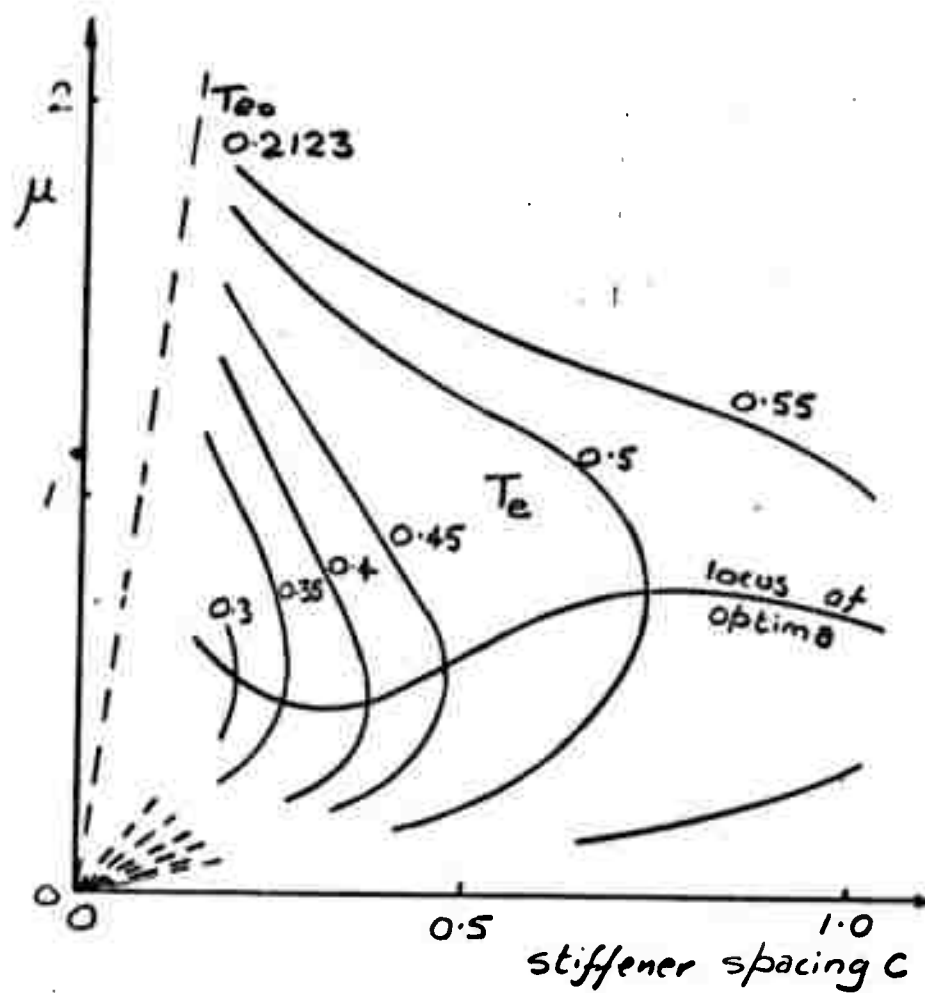spacing and stiffness.

(i) $R = 0.1$; $\eta = 1.0$.

FIGURE 3. Shear web equivalent thickness related to stiffener
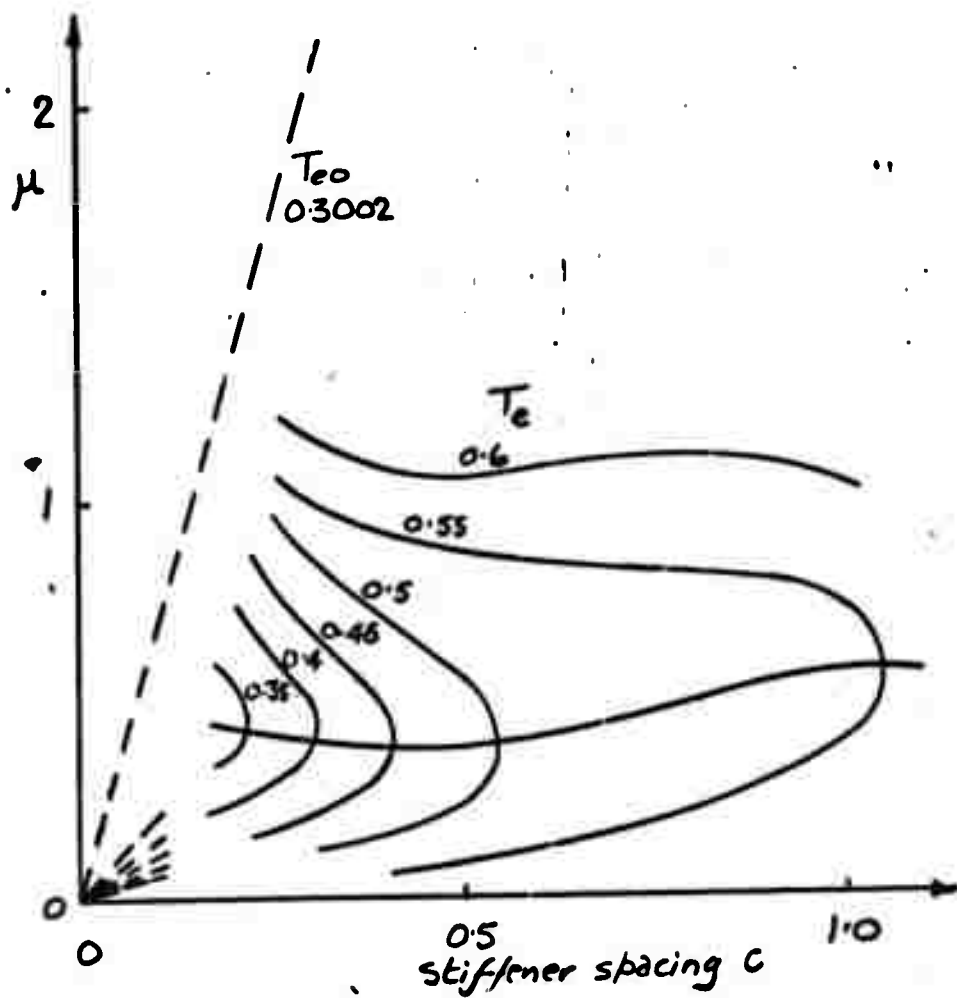spacing and stiffness
(ii) R = 0.5; η = 1.0.

FIGURE 4. Shear web equivalent thickness related to stiffener
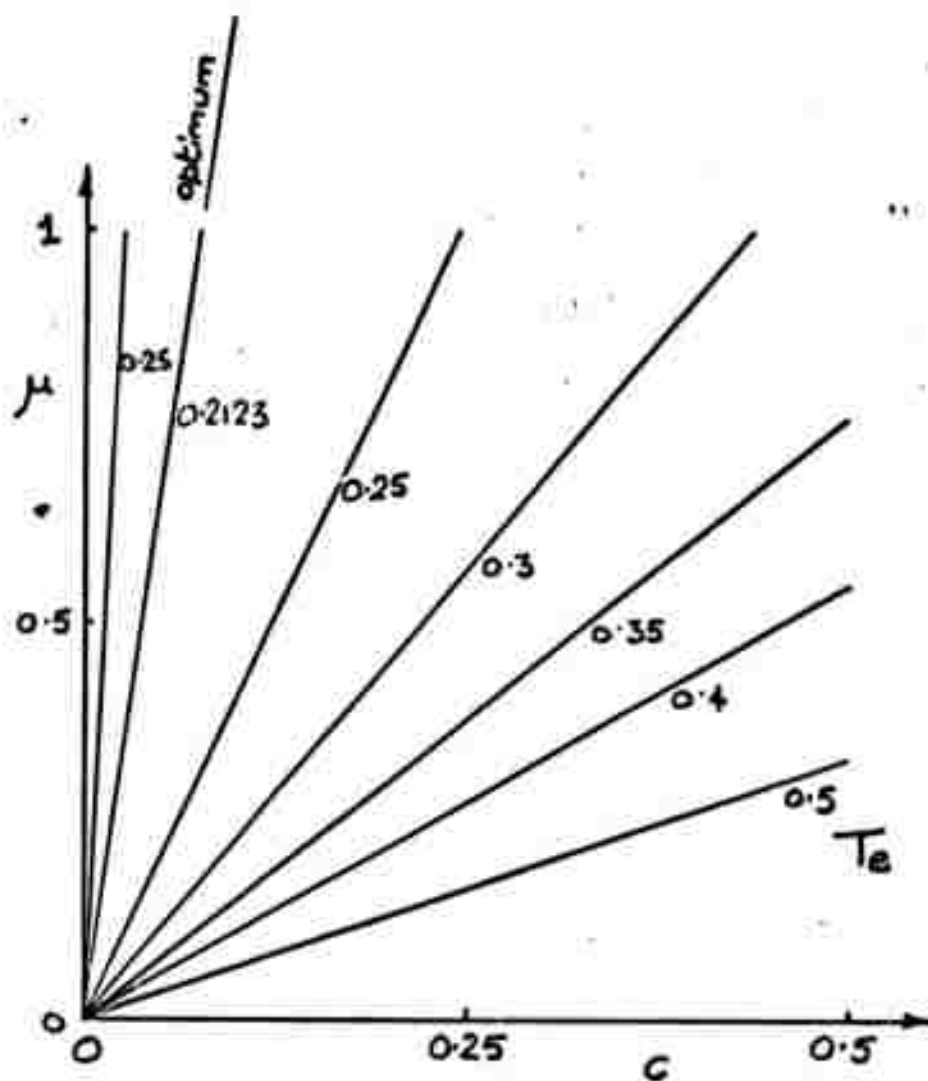spacing and stiffness
(iii) R = 1.0; η = 1.0.
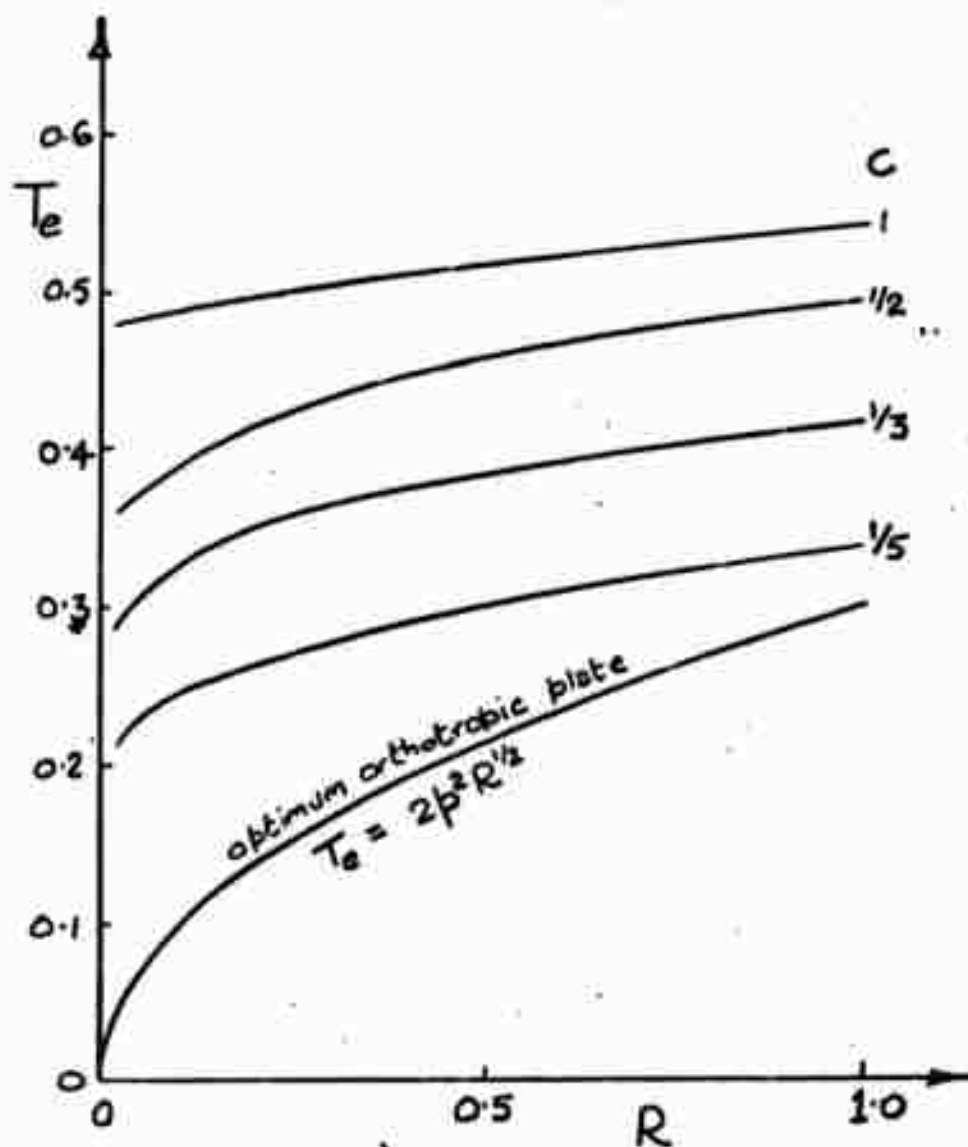
FIGURE 5. Orthotropic plate solution.
(R = 0.5, η = 1.0)

FIGURE 6.   Optimum shear web designs related to stiffener
spacing and load intensity ($\eta = 1.0$)
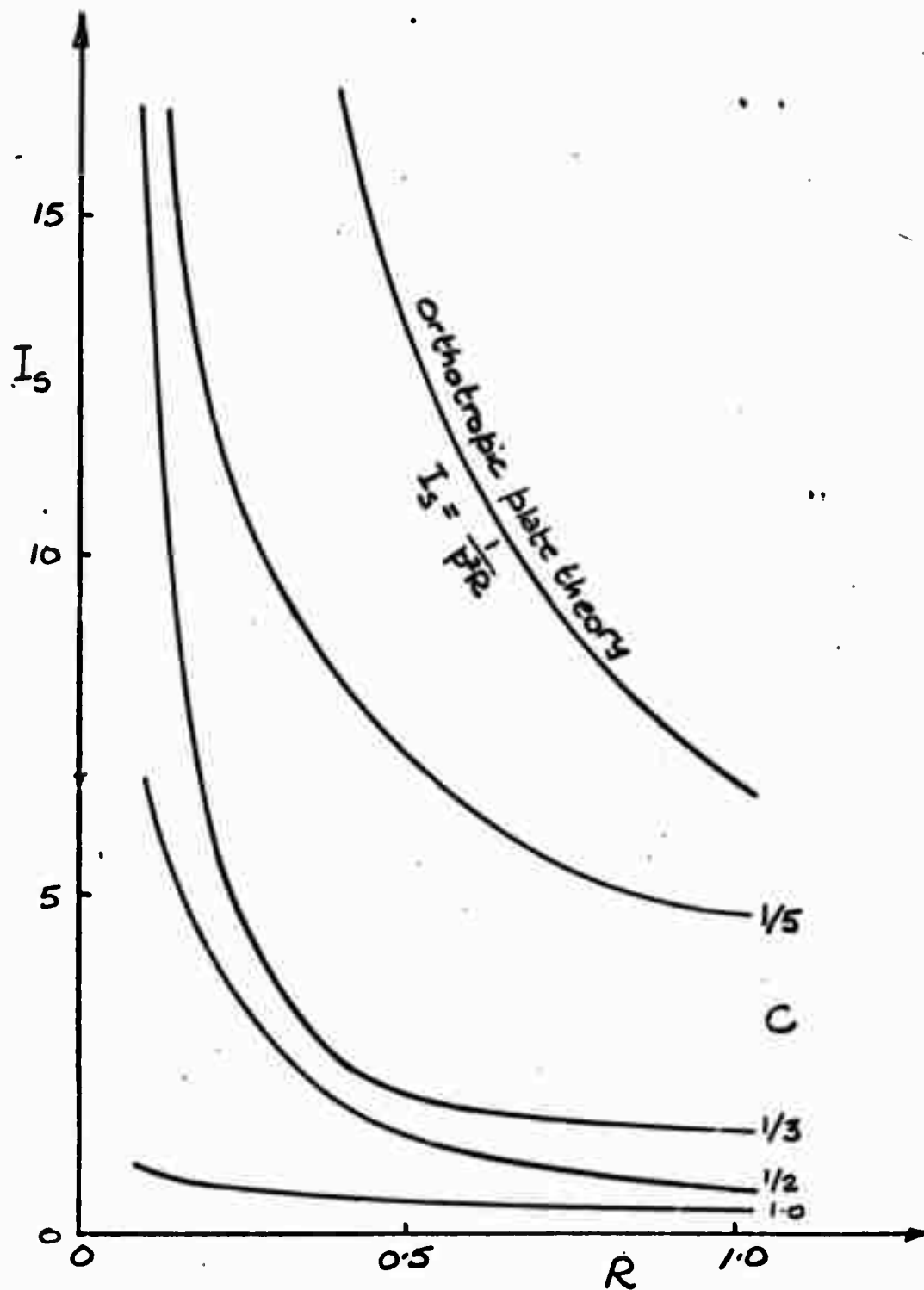
FIGURE 7. Stiffener bending stiffness required for optimum shear webs ($\eta = 1.0$)

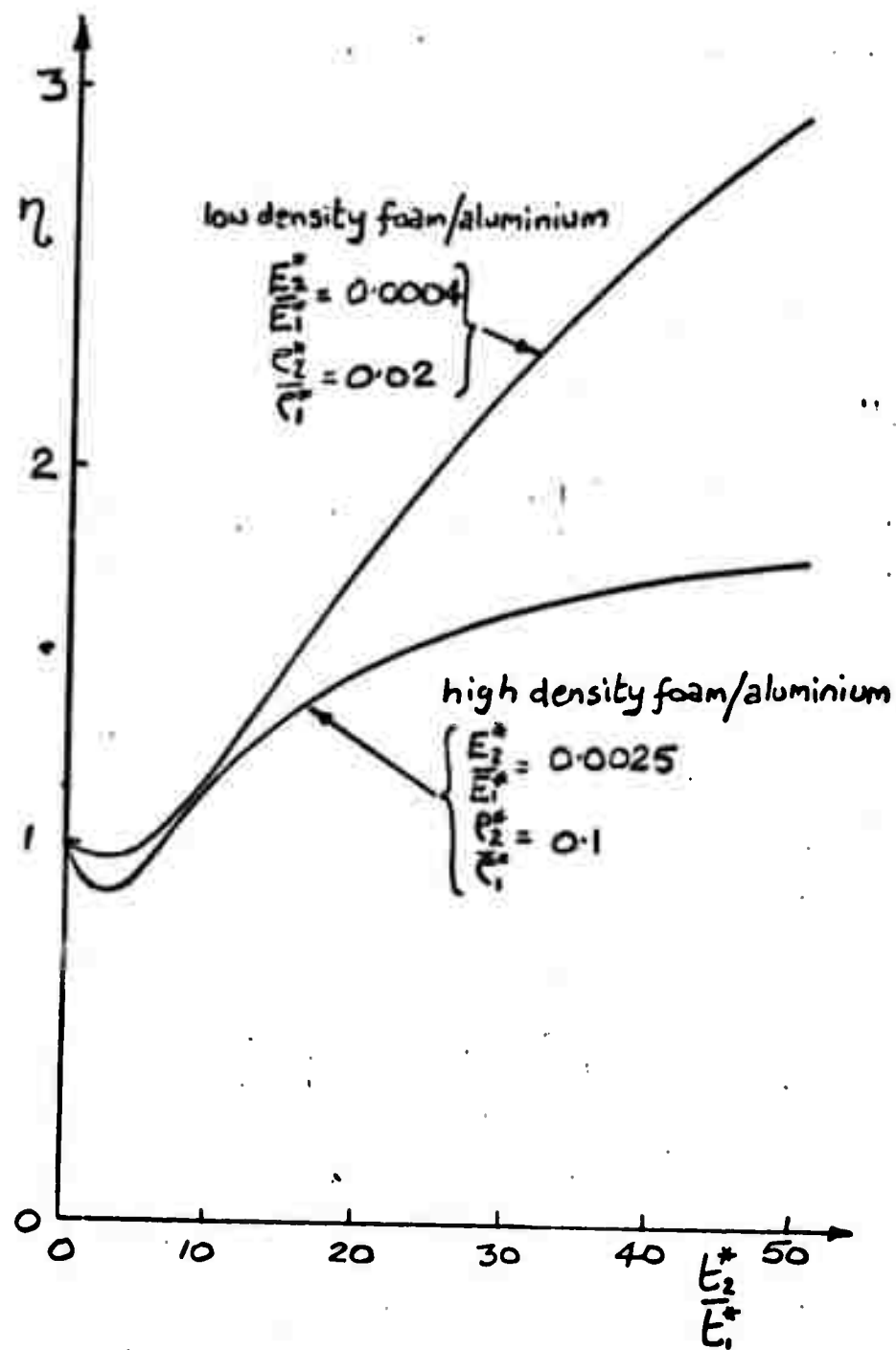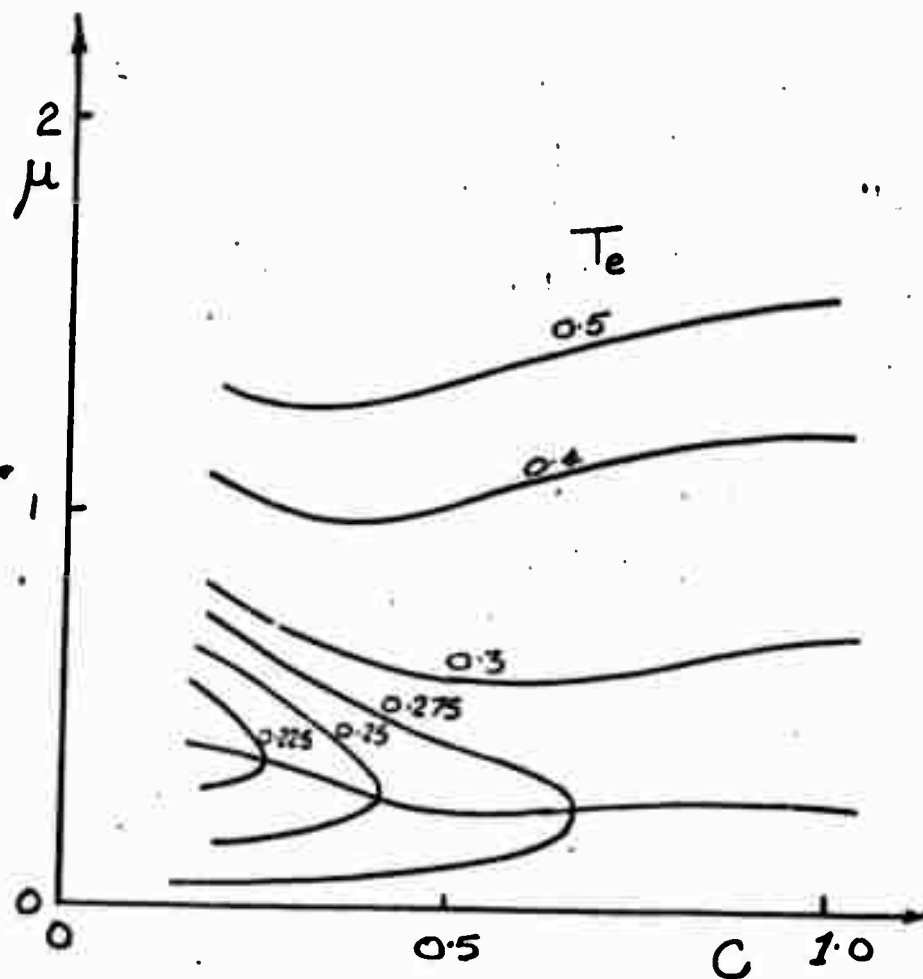FIGURE 8. Two layer plate efficiencies.

FIGURE 9.   Shear web designs with high efficiency skins
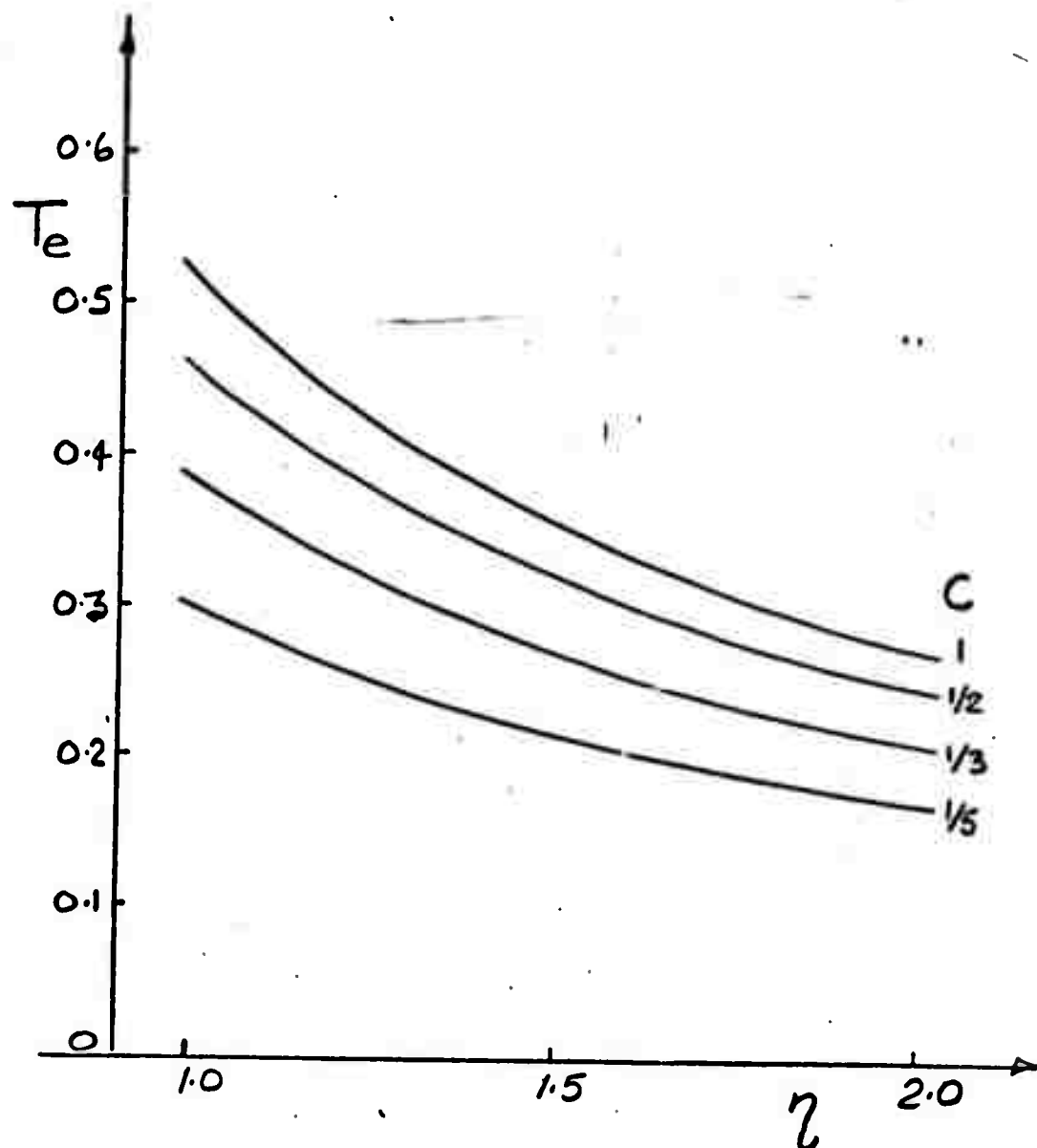   (R = 1.0, η = 2.0)

FIGURE 10.   Shear web equivalent thickness related to skin
efficiency. (R = 0.5)